Physics and Math of Music — Day 3 — Strings

Peter Folk (pfolk@uni) and Paul Grayson (pgrayson@uni)

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Strings are oscillators

So far we have only seen one example of an oscillator, the pendulum. Today you will learn about another way to make an oscillator — with a string. Oscillators made out of strings work just like pendulums, so we can do all the same things with them. However, there *is* one difference:

Strings have *harmonics*

An oscillator has one natural frequency, and will not vibrate at any other. A string, however, can vibrate in many different ways. Here are some of them: The second vibration has twice the frequency of the first; the

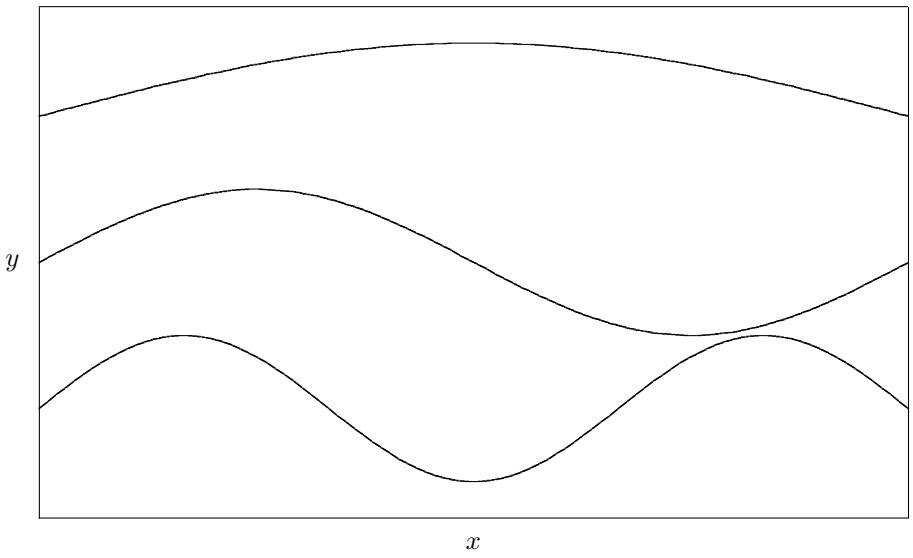

Figure 1: Different harmonics of a vibrating string

third has three times the frequency, and so on. See if you can make all of these using a slinky as your string! Next time you are talking on the phone, try this with the phone cord! How many harmonics *can* you make?

Since strings can oscillate at many different frequencies, it is a good idea to think of them as many different oscillators put together — just like our ruler with many pendulums.

We can calculate the frequencies of a string

We can calculate the natural frequencies of a string, using a formula very similar to the one for pendulums. This time, however, the frequencies depends not just on the length L, but also on the mass of the string m

and the tension T (the force you are pulling with to stretch it out):

$$f = 1 \cdot \frac{1}{2}\sqrt{\frac{T}{mL}} \ , \ \ 2 \cdot \frac{1}{2}\sqrt{\frac{T}{mL}} \ , \ \ 3 \cdot \frac{1}{2}\sqrt{\frac{T}{mL}} \ , \ ...$$

The most important thing here is that 1, 2, 3, ... relationship. Compare this to the pendulum formula and notice that there is a square root with some kind of force in the numerator and some kind of length in the denominator in both formulas!

Strings can resonate too

Just like a pendulum, a string can resonate. If you push it with a frequency near any one of the natural frequencies, it will start vibrating at that frequency. If you send in a wave made up of two frequencies near two different natural frequencies of the string, what do you think will happen? (Hint: remember what happens when two people blow on two pendulums at their two natural frequencies!)

From yesterday: make any wave out of sine waves

Here's a picture (which I stole) that shows how you can make a square wave by adding a bunch of sine waves together:

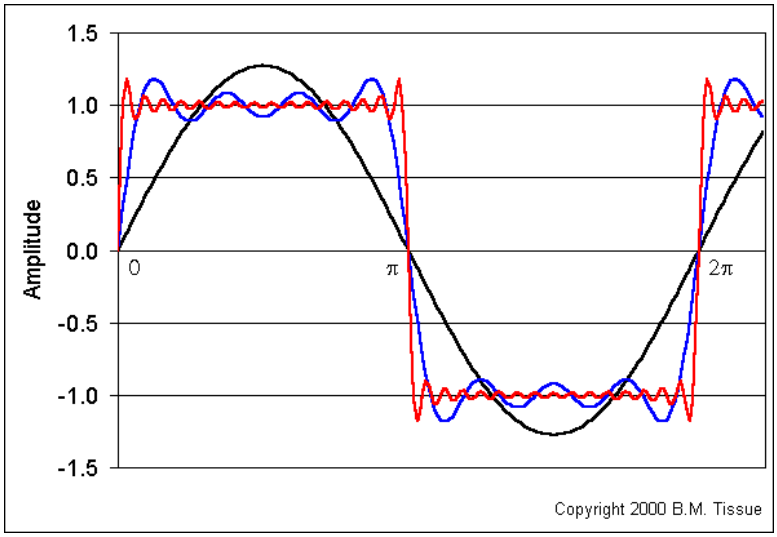

Figure 2: One, four, and sixteen sine waves added together to approximate a square wave. It takes a lot of sine waves, but it can be done!