Physics and Math of Music — Day 1 — Vibrations

Peter Folk (`pfolk@uni`) and Paul Grayson (`pgrayson@uni`)

Tuesday, February 5, 2002

Things vibrate with sine waves

Many things, when bumped, plucked, or shaken, start vibrating back and forth. For example, here is a weight hanging from a string:

Figure 1: A very simple pendulum.

If we give it a push, it will swing back and forth for a long time. Using a paintbrush and a long piece of paper, we can trace the motion of the weight. It follows a sine wave:

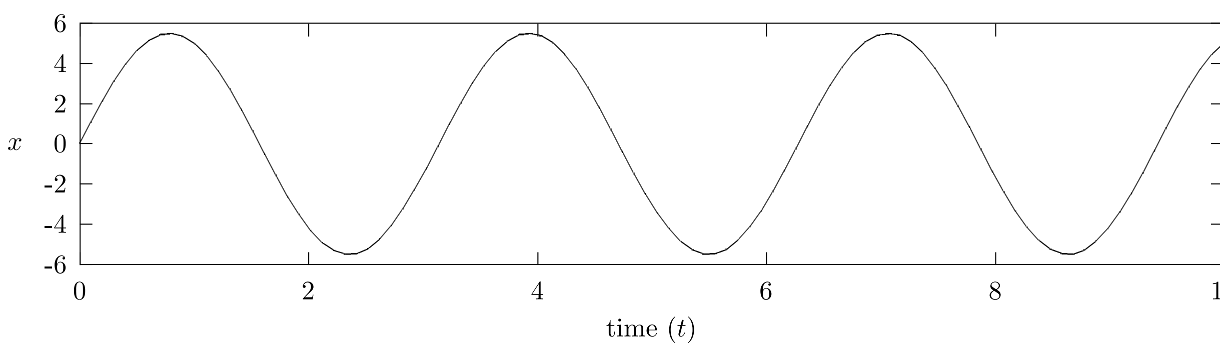

Figure 2: Plot of a simple sine wave: $x = A\sin(2\pi f t)$.

Things have natural frequencies

The pendulum swings back and forth at the same rate no matter how hard you hit it[1]. For example, if it swings back and forth 10 times per second, then $f = 4/\text{s}$, and f will be $4/\text{s}$ even when you swing it faster

[1] as long as you don't hit it *too* hard!

or slower. That's why pendulums are used for making clocks — they keep very good time. To calculate the natural frequency of a pendulum, all we need to know is L, its length:

$$f = \frac{1}{2\pi}\sqrt{\frac{9.8}{L/1\text{m}}}/\text{s}\,.$$

So, for example, if $L = 1\text{m}$, we get

$$f = \frac{1}{2\pi}\sqrt{\frac{9.8}{1\text{m}/1\text{m}}}/\text{s} = \frac{1}{2\pi}\sqrt{\frac{9.8}{1}} = \frac{1}{2\pi}\sqrt{9.8} = 0.5/\text{s}\,.$$

That means a one-meter pendulum takes 2 seconds to complete a full swing. Here's an important note: if you want to test our pendulum on another planet, you'll have to change the 9.8 — that number represents the strength of gravity.

Things resonate

If we give the pendulum lots of little kicks at its natural frequency, it will start moving faster and faster. This is called *resonance*, and it occurs everywhere. You know a lot about resonance if you have ever used a swing: you have to pump your legs at just the right frequency. If you are a little bit off, the swing doesn't move very much!